03/23/2017

Qualifying exam (工程數學)

1. Describe D.E. in the form of $y''+\frac{1-2a}{x}y'+(b^2c^2x^{2c-2}+\frac{a^2-p^2c^2}{x^2})y=0$ and the solution will be $y=x^a[c_1J_p(bx^c)+c_2Y_p(bx^c)]$. If p is not an integer, then Y_p can be replaced by J_{-p}

(a) Solve $9x^2y''+9xy'+(x^6-36)y=0$ (5%)

(b) Solve $x^2y''+4xy'+(x^2+2)y=0$ (5%)

2. A differential equation of the form $y'+p(x)y=f(x)y^n$, n is a real number, is called a ***Bernoulli's*** Equation. Please solve the following differential equation.

$$y'+\frac{1}{1+x}y=-(1+x)y^4,\quad y(0)=-2 \quad (15\%)$$

3. Solve the Cauchy-Euler differential equation (15%)

$$x^2y''+xy'-y=\ln x$$

4. Use **Laplace Transform** to solve the given integral function (10%)

$$f(t)+2\int_0^t f(\tau)\cos(t-\tau)d\tau=4e^{-t}+\sin t$$

5. Evaluate $\iint_R (x^2+y^2)^{-3}\,dA$, where R is the region bounded by the circles $x^2+y^2=2x$, $x^2+y^2=4x$, $x^2+y^2=2y$, $x^2+y^2=6y$. (15%)

6. If the matrix **A** can be diagonalized, then $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}=\mathbf{D}$ or $\mathbf{A}=\mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. (15%)

Show that $e^{\mathbf{A}t}=\mathbf{P}e^{\mathbf{D}t}\mathbf{P}^{-1}$

7. Solve the boundary-value problem (20%)

$$\frac{\partial^2u}{\partial x^2}+\sin x=\frac{\partial u}{\partial t},0<x<\pi,t>0$$

$$u(0,t)=400, u(\pi,t)=200, t>0$$

$$u(x,0)=400+\sin x, 0<x<\pi.$$