PRODUCTION ECONOMICS

Production economics is concerned with issues and problems in engineering economy and investment analysis that are specifically relevant to the production function.

I. Interest Factors and Notations

1. Single Payment Compound Amount Factor (SPCAF): The general equation for calculating the future equivalent of some present value P, can be determined by
   \[ F = P (1 + i)^n \]
   Where \( i \) is annual interest rate
   Notation of SPCAF = \((F/P, i\%, n)\)

2. Single Payment Present Worth Factor (SPPWF)
   \[ SPPWF = (P/F, i\%, n) = 1/(1 + i)^n \]

3. Capital Recovery Factor (CRF): Make uniform annual payments at the end of n year.
   \[ CRF = (A/P, i\%, n) = \left[ i \left(1 + i \right)^n \right]/\left[(1 + i)^n - 1\right] \]

   \[ USPWF = (P/A, i\%, n) = \left[ (1 + i)^n - 1 \right]/\left[i \left(1 + i \right)^n \right] \]

5. Sinking Fund Factor (SFF): We want to put aside a certain sum of money at the end of each year so that after n year, the accumulated fund, with interest compounded, will be worth F.
   \[ SSF = (A/F, i\%, n) = i/\left[(1 + i)^n - 1\right] \]

6. Uniform Series Compound Amount Factor (USCAF): It is desired to know how much money has accumulated after n year of uniform annual payments at interest rate.
   \[ USCAF = (F/A, i\%, n) = \left( (1 + i)^n - 1 \right)/i \]

II. Subjects

b. Production cost
c. Breakeven analysis.

A. Methods of Evaluating Investment Proposals for Manufacturing

1. Payback Period Method
   Uses the simple concept that that net revenues derived from an investment should pay back the investment in a certain period of time.

   \[ NACF = \text{Net annual cash flow} \]
   \[ IC = \text{initial cost} \]
   \[ \text{Payback period } n = IC/NACF \]

Ex. a. (Pay-Back) A new product machine tools $85,000 installed and is expected to generate revenues of $55,000 per year for 7 years. It will cost $30,000 per year to operate the machine. At the end of 7 years, the machine will be scrapped at zero salvage value. Determine the payback period for this investment.
\[ NACF = 55,000 - 30,000 = 25,000 \]
\[ IC = 85,000 \]

**Payback period** \( n = IC/NACF = 3.4 \) years

**Real life situations**: The net annual cash flows will not be equal year after year.

\[ 0 = IC + \sum (NACF) \]

2. **Present Worth (PW) Method**
   - Uses the equivalent present value of all current and future cash flows to evaluate the investment proposal.
   - The future cash flow are converted into their present worth by using the appropriate interest factors.
   - The interest rate is decided in advance and represent the rate-of-return criterion that the company is using to evaluate its investment opportunities.

   \[ PW > 0, \rightarrow \text{The return from the project exceeds the rate-of-return criterion}. \]

   \[ PW < 0, \rightarrow \text{The project does not meet the rat-of-return criterion}. \]

**Example**: The data from payback method. Assume that the company considering the investment uses a rate-of-return criterion of 20%. Determine the equivalent present worth of the proposal.

\[ PW = -85,000 + 55,000 \left(P/A, 20\%, 7\right) - 30,000 \left(P/A, 20\%, 7\right) \]
\[ = -85,000 + 25,000 \left(3.6046\right) \]
\[ = $5,115 \]

\[ PW > 0, \text{Project is therefore meritorious}. \]

3. **Uniform Annual Cost (UAC) Method**
   - Converts all current and future cash flows to their equivalent uniform annual costs using he given rate of return.

   \[ UAC > 0, \rightarrow \text{project exceeds the criterion} \]

**Example**: Data from payback method

\[ UAC = -85,000 \left(A/P, 20\%, 7\right) + 55,000 - 30,000 \]
\[ = -85,000 \left(0.2774\right) + 25,000 \]
\[ = $1,421 \]

\[ \therefore \text{Actual rate of return is greater than 20\%} \]

4. **Rate-of-Return Method (Return-on-Investment (ROI))**
   - PW and UAC are found the worth of project.
   - ROI: goes slightly beyond the PW and UAC method by actually calculating the rate-of-return that is provided by the investment.

**Example**: Data from payback method

Either the PW method or the UAC method can be used to establish

\[ UAC = -85,000 \left(A/P, i, 7\right) + 55,000 - 30,000 \]

Set \( UAC = 0, \)

\( \left(A/P, i, 7\right) = 25,000 / 85,000 = 0.2941 \]

From table \( i = 20\% \sim 25\% \)

by interpolation \( i = 22.15\% \)

5. **Comparison of Investment Alternatives**
Ex b. (comparison) Two production methods, one manual & the other automated, are to be compared using the UAC method. The data for the manual method are the same data above. For the automated method, IC = $150,000 the annual operating cost = $5,000, and the service life is expect to be 5 years. In addition, the equipment associated with this alternatives will have a salvage value = $15,000 at the end of the 5 years. Revenues from either alternative will be $55,000 per year. A 20% rate of return is to be used as the criterion.

**UAC method:**

Manual production method, UAC is obtained from

\[
UAC = -85,000 \times (A/P, 20\%, 7) + 55,000 - 30,000
\]

\[
= -85,000 \times (0.2774) + 25,000
\]

\[
= $1,421
\]

Automated production method

\[
IC = 150,000
\]

\[
UAC = -150,000 \times (A/P, 20\%, 5) - 5,000 + 55,000 + 15,000 \times (A/F, 20\%, %)
\]

\[
= -150,000 \times (0.33438) - 5,000 + 55,000 + 15,000 \times (0.13438)
\]

\[
= $1,859
\]

Each method has its relative advantage and disadvantage

1. Payback method: easy to comprehend but not incorporate the concept of time value of money into its value (quick performance measure for investment)
2. Present worth method: also easy to understand and it does include interest rates in the evaluation.
3. UAC method: convenient to use when the service lives of the alternatives are different.
4. Rate-of-return: provide a value for the expected return on the investment.
5. Comparison of investment alternatives: the most practical uses.

**B. Production Cost (Costs in Manufacturing):** Can be divided into four major categories:

1. Fixed Costs
2. Variable Costs
3. Overhead Costs
   a. Factory Overhead
   b. Corporate Overhead
4. Cost of Equipment Usage

1. **Fixed Costs (FC)**
   - Is one that is constant for any level of production output, these include cost of the factory building, insurance, property taxes, and cost of production equipment.
   - Those items that are capital investment can be converted to their equivalent uniform annual costs by the methods which we discussed before.

2. **Variable Costs (V_{unit}, TVC_{Total})**
   - Is one that increase as the level of production increases, such as direct labor costs, raw materials, and electrical power to operate the production machines.
3. Overhead Costs

- Are all the other costs associated with running a manufacturing firm. This can be divided into two categories.

A. Factory overhead:

Includes the cost of operating the factory other than direct labor and materials.

B. Corporate overhead cost:

Is the cost of running the company other than its manufacturing activities.
Such as: Corporate executives, Sales personnel, Accounting department, R&D, etc.

Overhead analysis: Can be allocated according to a number of different bases, including direct labor cost, direct labor hour, space, space, material cost, and so on.

Factory Overhead Rate = \frac{\text{Total cost of operating a plant} \ - \ \text{Direct labor cost}}{\text{Direct Labor Cost}}

4. Cost of Equipment Usage

- The overhead rates as we have developed is that they are based on direct labor cost alone. Therefore, a machine operator who runs an old and small engine lathe will be cost at the same overhead rate as the operator who runs a modern NC machining center. But this could be argued by capital investments, the time on the automated machine should be valued at a higher rate.
- If difference between rates of different production machines are not recognized, manufacturing will not be accurately measured by the overhead rate structure.
- To overcome this difficulty, production costs divided into a. Direct labor, b. Machine cost.

Ex c. (Cost of equipment usage) The determination of an hourly rate for given work center can best be illustrated by means of given as follows,

\begin{align*}
\text{direct labor rate} &= \$7.0/h \\
\text{applicable labor factory overhead rate} &= 60\% \\
\text{capital investment in m/c} &= \$100,000 \\
\text{service life} &= 8 \text{ years} \\
\text{salvage value} &= 0 \\
\text{applicable m/c factory overhead rate} &= 50\% \\
\text{rate of return usage} &= 15\% \\
\end{align*}

The m/c is operated one 8 hours shift/day, 250 days/year. Determine the appropriate hourly cost for this work m/c system.

Solution:

\begin{align*}
\text{The labor cost per hour is } 7 \ (1 + 0.6) &= \$11.2/h \\
\text{The machine cost must first be annualized} \\
\text{UAC} &= 100,000 \times (A/P, 10\%, 8) \\
&= 100,000 \times (0.18744) \\
&= \$18,744/\text{yr} \\
\text{Number of hours} 8 \times 250 &= 2000 \text{ hr/yr} \\
\text{Machine cost:} 8,744/2,000 &= \$9.37/h \\
\text{Machine overhead and machine cost:} 9.37 \ (1 + 0.5) &= \$14.06/h \\
\text{Therefore, total work center rate} &= 11.20 + 14.06 = \$25.26/h
\end{align*}
5. Total Cost (TC)
   \[ TC = TVC + FC \]

C. Break-Even Analysis
   • Break-even analysis can be used for either of two main purposes:

1. Profit Analysis: The break-even chart shows the effect of changes in output on costs and revenues.
2. Production method cost comparison: Shows the effect of changes in output level on the costs of two (or more) different methods of production.

Ex d. (Profit Analysis) A annually operated production m/c costs $66,063. It will have a service life of 7 years with an anticipated salvage value of $5,000 at the end of its life. The m/c will be used to produce one type of part at a rate of 20 units/h. The annual cost to maintain the m/c is $2,000. A m/c overhead rate of 15% is applicable to capital cost and maintenance labor to run the m/c costs $10.0/h and the applicable overhead rate is 30%. Determine the profit break-even point if the value added is $1.0/unit and the rate of return criterion is 20%.

Solution:
   • **Variable cost** is labor cost, including applicable overhead
     \[ \frac{10/h \times (1 + 30%)}{20 \text{ unit/h}} = 0.65/\text{unit} \]
     Let \( Q \) be the annual level, then the variable cost as a function of \( Q \) is \( 0.65Q \)
   • **Fixed cost**
     \[ UAC = 66,063 \times (A/P, 20\%, 7) + 2,000 - 5,000 \times (A/F, 20\%, 7) \]
     = $19,939
     Adding the 15% overhead, the fixed cost = $22,930
   • The sum of the fixed and variable cost
     Total Cost = $22,930 + 0.65Q
     Revenues as a function of \( Q \) are the product of value added per unit multiplied by \( Q \).
     Revenues = $1.00Q
   • The break-even point
     Profit = 1.00Q – 22,930 – 0.656Q = 0
     \( Q = 65,514 \text{ units/yr} \)

Production Method Cost Comparison

Ex e. (The cost break-even Analysis) The alternative is an automated m/c, costing $125,000, but capable of a production rate of 50 units/h. Its service life is 5 years with no salvage value at the end of that time. Annual maintenance will cost $5,000. One-third of one operator costing $12.0/h will be required to run the m/c. The overhead rate and rate of return used in last example are applicable. Determine the break-even point for the automated and manual methods of production.

Solution: for the automated machine
   • **Variable cost**
     \[ \frac{($12/h)(1/3)(1+30\%)}{50\text{ pieces/h}} = 0.104/\text{unit} \]
• **Fixed cost with overhead added**
  \[125,000 \text{ (A/P, 20%, 5)} + 5,000 (1 + 15\%) = 53,820\]
  \[\text{Total Cost} = 53,820 + 0.104Q\]
• **Profit for automated**
  \[\text{Profit} = 1.0Q - 53,820 - 0.104Q = 0\]
  \[Q = 60,067\text{ units/yr}\]
• **The break-even point is represented by the intersection point for the two methods**
  \[53,820 + 0.104Q = 22,930 + 0.65Q\]
  \[Q = 56,575\text{ units/yr}\]
• **For manual corresponds to 2829 hours/yr**
• **For automated corresponds to 1131.5 hours/yr**

### Unit Cost of Production

To help decide between the alternatives, it is often useful to determine the unit cost of production.

**Unit Cost** = \[\frac{\text{Total Cost (TC)}}{\text{Number of unit product (Q)}}\]

• Because of the fixed portion of the cost of production, the unit cost will vary as a function of annual output **Q**.
• Annual output increases, the unit cost decreases.

\[C_{pc, \text{manual}} = \frac{22,930 + 0.65Q}{Q} = 0.65 + \frac{22,930}{Q}\]

\[C_{pc, \text{automated}} = \frac{53,820 + 0.104Q}{Q} = 0.104 + \frac{53,820}{Q}\]

• From these two \(C_{pc}\), the break-even point 56,575 units/yr

### Discussion

• From Example the number of annual hours of operation at the profit break-even point = 3276 hours, this is greater than the number of hours normally worked by one person/yea (40/week x 50 weeks/yr = 2000hours/yr). In this situation, how to improve the system and make profit is the mainly job.
  1. Achieved by using two machines – fixed costs may increased
  2. By using two shifts on one machine – second shifts probably be paid at higher rate.
  3. By working overtime by one production worker – higher rate at overtime.
**Break-even summary**

From above, we may summary the symbols and described in following:

- **Q** = number of units
- **FC** = fixed cost
- **V** = unit variable cost
- **TVC** = total variable cost = QV
- **TC** = total cost = TVC + FC
- **P** = unit selling price
- **R** = revenue = QP
- **Z** = profit = R − TC
- **B** = break-even volume = FC/(P − V)

And **P − V** = unit contribution

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**Cost of manufacturing lead time and work-in-process**

- Production consists of a series of separate manufacturing steps or operations.
- An operation time is required for each step, and that time has an associated cost, **C_o**.
- None-operation time includes material handling inspection, and storage, **C_no**.
- Set production time **T_p** = set up time + operation time.
- The cost associated with each processing step in the manufacturing sequence, 
  \[ C_o T_p + C_no \]
- The total cost invested in the part at the end of all operations, **C_pc**.
  \[ C_{pc} = C_m + \sum (C_o T_{pi} + C_{noi}) \]
  Where **i** indicate the sequence of operations.

\[ \sum (C_o T_{pi} + C_{noi}) \]

To simplified the process, assume **T_p** and **C_no** equal for all **n_m**

\[ C_{pc} = C_m + n_m (C_o T_{pi} + C_{noi}) \]

\[ \sum (C_o T_{pi} + C_{noi})/MLT \]

- Another two cost need to be considered: The cost of investing in WIP and the cost of storing the WIP
- Combining the interest rate, **i** and storage rate **s**, the holding cost \( h = i + s \), Total cost:
TC_{pc} = C_m + n_m (C_o T_p + C_{no}) + \int_{0}^{\text{MLT}} \left[ C_m + \frac{C_m + n_m (C_o T_p + C_{no})}{\text{MLT}} \right] h dt

\bullet \text{Let } n_m (C_o T_p + C_{no}) = C_1
And integrate
TC_{pc} = C_m + C_1 + (C_m + C_1/2) h (\text{MLT})
The holding cost/piece = (C_m + C_1/2) h (\text{MLT})

**Equipment Replacement Decisions**

\bullet Replace a piece of equipment before end of its economic life, based on their economic advantages and evaluated through investment analysis.

Example: Packaging equipment was purchased three years ago at a cost of $50,000. The operating costs for this equipment are estimated to be $8,000 a year. The equipment was expected to last for eight years at which time it was estimated it could be sold for $10,000. The annual revenue due to the equipment is estimated to be $25,000. Now, after three years, a new system is available in the market which is far more efficient than the existing one. The life of this new equipment is 6 years, it costs $80,000, and has an estimated salvage value of $20,000. Because of the high efficiency of the new equipment its annual operating cost is only $5,000 and it produces a revenue of $35,000 a year. If the management decides to replace this equipment now, the old equipment can be sold to another manufacturer at a price of $25,000. There is also a possibility of using the old equipment in another department which packages less important products. If the old equipment is sold, similar equipment must be purchased for that department at a cost of $35,000. The company needs to decide whether the replacement should be made. The going rate of interest is 12%.

**Solution:**

\bullet Replace a piece of equipment before end of its economic life base on their economic advantages and evaluated through investment analysis.

New: Cost $80,000
Salvage $20,000
Life 6 years
Annual operating cost $5,000
Annual revenue $5,000

1. Replace $\rightarrow$ the old equipment can be sold $\rightarrow$ $25,000$
2. Not replace send to another department do less important product (if sold need buy similar equipment) $\rightarrow$ 35,000

\bullet Use graphic to represent

Cost $\rightarrow$

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Revenue
Salvage
Total Annual Cost = Annual Capital + Annual Op. Cost
= 80,000(A/P, 12%, 6) + 5,000
= 80,000(0.2432) + 5,000 = 24,456

Total Annual Revenue = Annual Revenue + Revenue due to Salvage value
= 35,000 + 20,000(A/F, 12%, 6)
= 35,000 + 20,000(0.1232) = 37,464

Net Annual Profit = Total Annual Revenue - Total Annual Cost = 13,008 \rightarrow \text{New}

- Max. Future profits – discuss what is the investment. Now, instead of 3 years ago for next five years.
- Sold old equipment \rightarrow 25,000

If (Case 1) Sold (25,000) \rightarrow \text{no Annual Cost, no salvage but need another investment}
(Case 2) Keep (35,000) \rightarrow \text{need Annual Op. Cost, and salvage}

Case 1.

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Total Annual Cost = 35,000(A/P, 12%, 5) + 8,000
= 35,000(0.2774) + 8,000 = 17,709

Total Annual Revenue = 25,000 + 25,000(A/P, 12%, 5)
= 25,000 + 25,000(0.2432) = 31,953

Net Annual Profit = 14,226

Case 2.

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Total Annual net Profit = 10,000(A/F, 12%, 5) + 25,000 – 17709 = 8,865

New Investment during the Process Year

Example: A company is planning to install a new material handling system. There are two alternatives available. Alternative A requires an initial investment of $300,000 and is expected to be operable for the next eight years without requiring any major repair. The annual operating cost for this system is estimated at $25,000 a year, and it is expected that some of the equipment can be sold at the end of the eighth year for $40,000. It is estimated that installing this system will result in an annual savings of $120,000 for the company.

Alternative B calls for an initial investment of only $150,000 and is expected to last for five years. It is expected that at the end of the second year a new set of accessory equipment will be available for this system which has to be added at a cost of $100,000. However, adding this new equipment will reduce the annual operating cost from $30,000 a year for the first two years to $20,000 for the remaining years. The salvage value for this alternative is $50,000. This alternative will produce an annual savings of $115,000. Use a 15% interest rate in the computations.

Solution:
New investment during the process year

**Alternative A.**

\[
\text{Total Annual Cost} = 300,000 \times (A/P, 15\%, 8) + 25,000 = 91,870
\]

\[
\text{Total Annual Saving} = 40,000 \times (A/F, 15\%, 8) + 120,000 = 122,916
\]

\[
\text{Net Annual Saving} = 31,046 \rightarrow \text{for Alternative A.}
\]

**Alternative B.**

1. **Present worth of all investments**

   \[
   \text{PW of 100,000 (IC) = 100,000 \times (P/F, 15\%, 2) = 75,610}
   \]

   Total present worth of capital = 150,000 + 75,610 = 225,610

   Annual Capital Cost = 225,610 \times (A/P, 15\%, 5) = 67,299

   \[\rightarrow \text{Annual Op. Cost is not the same, Therefore, convert these to the PW}\]

   a. \[30,000 \times (P/F, 15\%, 1) = 26,888\]
   b. \[30,000 \times (P/F, 15\%, 2) = 22,683\]
   c. \[30,000 \times (P/F, 15\%, 3) = 13,150\]
   d. \[30,000 \times (P/F, 15\%, 4) = 11,436\]
   e. \[30,000 \times (P/F, 15\%, 5) = 9,944\]
   Sum of a ~ e = 83,301

   Equivalent Annual Operating Cost = 83,301 \times (A/P, 15\%, 5) = 24,849

   Total Annual Cost = 67,299 + 24,849 = 92,148

   Total Annual Revenue = 115,000 + 50,000 \times (A/F, 15\%, 5) = 122,415

   Net Annual Profit = 122,415 – 92,148 = 30,267 \rightarrow \text{for Alternative B}

Compare alternative A and B, A is the better choice.
### 10%

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