Internal Forced Convection: Heat Transfer Correlations

Chapter 8
Sections 8.4 through 8.6
8.4 Laminar Flow in Circular Tubes

- **Laminar Flow in a Circular Tube:**

  The local Nusselt number is a constant throughout the fully developed region, but its value depends on the surface thermal condition.

  - Uniform Surface Heat Flux \((q'_s)\):

    \[
    Nu_D = \frac{hD}{k} = 4.36
    \]

    \(q'_s = \text{常數}\)

  - Uniform Surface Temperature \((T_s)\):

    \[
    Nu_D = \frac{hD}{k} = 3.66
    \]

    \(T_s = \text{常數}\)
The steady flow energy balance for a cylindrical shell element can be expressed as:

\[ \dot{m}c_p T_x - \dot{m}c_p T_{x+dx} + q_r - q_{r+dr} = 0 \]

Substituting \( \dot{m} = \rho u A_c = \rho u (2\pi r dr) = 質量流率 \) and dividing by \( 2\pi dr dx \) gives, after rearranging

\[ \rho c_p u \left( \frac{T_{x+dx} - T_x}{dx} \right) = -\left( \frac{1}{2\pi r dx} \right) \left( \frac{q_{r+dr} - q_r}{dr} \right) \]
Or

\[ u \frac{\partial T}{\partial x} = -\left( \frac{1}{2\rho c_p \pi r dx} \right) \frac{\partial q}{\partial r} \]

- Since Fourier's Law

\[ \frac{\partial q}{\partial r} = \frac{\partial}{\partial r} \left( -k(2\pi r dx) \frac{\partial T}{\partial r} \right) = -2\pi k dx \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \]

代回得到(能量平衡公式 – Energy Equation)

\[ u \frac{\partial T}{\partial x} = \alpha \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \quad ; \quad \text{其中} \quad \alpha = \frac{k}{\rho c_p} \quad (8-48) \]
Constant Surface Heat Flux (完全發展區域)

\[ \frac{\partial}{\partial x} \left( \frac{T_s - T}{T_s - T_m} \right) = 0 \]

Hence,
\[ \frac{1}{T_s - T_m} \left( \frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x} \right) - \frac{T_s - T}{(T_s - T_m)^2} \left( \frac{\partial T_s}{\partial x} - \frac{\partial T_m}{\partial x} \right) = 0 \]

or,
\[ \frac{\partial T}{\partial x} = \frac{dT_s}{dx} - \left( \frac{T_s - T}{T_s - T_m} \right) \frac{dT_s}{dx} + \left( \frac{T_s - T}{T_s - T_m} \right) \frac{dT_m}{dx} \]

(i) For constant surface heat flux case:
\[ q_s'' = h(T_m - T_s) = \text{constant} \]

\( (T_m - T_s) \) must also be a constant. So,
\[ \frac{dT_m}{dx} = \frac{dT_s}{dx}, \text{ substitute it back,} \]

it yields
\[ \frac{\partial T}{\partial x} = \frac{dT_s}{dx} \]

\[ \frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{q_s''(2\pi r_o)}{\dot{m} c_p} = \frac{q_s''(2\pi r_o)}{(\rho u_m \pi r_o^2) c_p} = \frac{2q_s''}{\rho u_m r_o c_p} = \text{constant} \]
• Substituting \( u \) and \( \left( \frac{\partial T}{\partial x} \right) \) into Eq. 8.48

\[
\frac{\partial T}{\partial x} = \frac{2q''_s}{\rho u_m c_p r_o} = \text{constant}
\]

\[
u(r) = 2u_m \left( 1 - \frac{r^2}{r_o^2} \right)
\]

\[
u \frac{\partial T}{\partial r} = \alpha \frac{\partial}{r \ dr} \left( r \frac{\partial T}{\partial r} \right)
\]

\[
\frac{4q''_s}{kr_o} \left( 1 - \frac{r^2}{r_o^2} \right) = \frac{1}{r \ dr} \left( r \frac{dT}{dr} \right)
\]

常微分方程式
Separating the variables and integrating twice

\[
T = \frac{q_s^\prime} {kR} \left( r^2 - \frac{r^4}{4r_o^2} \right) + C_1 r + C_2
\]

Boundary conditions

- Symmetry at \( r = 0 \): (i) \( \frac{\partial T(r = 0)}{\partial r} = 0 \) \( C_1 = 0 \)

- At \( r = r_o \): (ii) \( T(r = R) = T_s \) \( C_2 \)

\[
T = T_s - \frac{q_s^\prime r_o}{k} \left( \frac{3}{4} - \frac{r^2}{r_o^2} + \frac{r^4}{4r_o^2} \right)
\]

(8-57)
常數温度

\[ T_m = \text{平均流體温度} = \frac{2}{u_m r_o^2} \int_0^R T(r) u(r) r dr \]

代入積分

\[ T_m = T_s + \frac{11}{24} \frac{q_s r_o}{k} \]

\[ q_s'' = h(T_s - T_m) \]

\[ h = \frac{24 k}{11 r_o} = \frac{48 k}{11 D} = 4.36 \frac{k}{D} \]

Constant heat flux (circular tube, laminar)

\[ Nu = \frac{hD}{k} = 4.36 = \text{常數} \]
Constant Surface Temperature
(circular tube, laminar)

\[ Nu = \frac{hD}{k} = 3.66 \]  \hspace{1cm} (8-55)

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Example 8.4

One concept used for solar energy collection involves placing a tube at the focal point of a parabolic reflector and passing a fluid through the tube.

The net effect of this arrangement may be approximated as one of creating a condition of uniform heating at the surface of the tube. That is, the resulting heat flux to the fluid \( q_s'' \) may be assumed to be a constant along the circumference and axis of the tube. Consider operation with a tube of diameter \( D = 60 \) mm on a sunny day for which \( q_s'' = 2000 \) W/m\(^2\).

1. If pressurized water enters the tube at \( \dot{m} = 0.01 \) kg/s and \( T_{m,i} = 20^\circ\text{C} \), what tube length \( L \) is required to obtain an exit temperature of \( 80^\circ\text{C} \)?

2. What is the surface temperature at the outlet of the tube, where fully developed conditions may be assumed to exist?
Water

\( \dot{m} = 0.01 \text{ kg/s} \)

\( D = 60 \text{ mm} \)

\( T_{m,i} = 20^\circ \text{C} \)

\( q_s'' = 2000 \text{ W/m}^2 \)

\( T_{s,o} \)

\( T_{m,o} = 80^\circ \text{C} \)

\( L \)
**Properties:** Table A.6, water ($\overline{T}_m = 323$ K): $c_p = 4181$ J/kg · K. Table A.6, water ($T_{m,o} = 353$ K): $k = 0.670$ W/m · K, $\mu = 352 \times 10^{-6}$ N · s/m$^2$, $Pr = 2.2$.

**Analysis:**

1. For constant surface heat flux, Equation 8.38 may be used with the energy balance, Equation 8.34, to obtain

\[
A_s = \pi DL = \frac{\dot{m}c_p(T_{m,o} - T_{m,i})}{q_s''}
\]

\[
L = \frac{\dot{m}c_p}{\pi Dq_s''} (T_{m,o} - T_{m,i})
\]

Hence

\[
L = \frac{0.01 \text{ kg/s} \times 4181 \text{ J/kg} \cdot \text{K}}{\pi \times 0.060 \text{ m} \times 2000 \text{ W/m}^2 (80 - 20)\text{°C}} = 6.65 \text{ m}
\]

2. The surface temperature at the outlet may be obtained from Newton’s law of cooling, Equation 8.27, where

\[
T_{x,o} = \frac{q_s''}{h} + T_{m,o}
\]
To find the local convection coefficient at the tube outlet, the nature of the flow condition must first be established. From Equation 8.6,

\[ Re_D = \frac{4 \dot{m}}{\pi D \mu} = \frac{4 \times 0.01 \text{ kg/s}}{\pi \times 0.060 \text{ m} \times 352 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 603 \]

Hence the flow is laminar. With the assumption of fully developed conditions, the appropriate heat transfer correlation is then

\[ Nu_D = \frac{h D}{k} = 4.36 \]

and

\[ h = 4.36 \frac{k}{D} = 4.36 \frac{0.670 \text{ W/m} \cdot \text{K}}{0.06 \text{ m}} = 48.7 \text{ W/m}^2 \cdot \text{K} \]

The surface temperature at the tube outlet is then

\[ T_{s,o} = \frac{2000 \text{ W/m}^2}{48.7 \text{ W/m}^2 \cdot \text{K}} + 80^\circ \text{C} = 121^\circ \text{C} \]

Comments: For the conditions given, \((x_{sd}/D) = 0.05 Re_D Pr = 66.3\), while \(L/D = 110\). Hence the assumption of fully developed conditions is justified. Note, however, that with \(T_{s,o} > 100^\circ \text{C}\), boiling may occur at the tube surface.
Example 8.5

In the human body, blood flows from the heart in a series of branching blood vessels having successively smaller diameters. The capillaries are the smallest blood vessels. In developing the bioheat equation (Section 3.7), Pennes assumed that blood enters the capillaries at the arterial temperature and exits at the temperature of the surrounding tissue. This problem tests that assumption [5,6]. The diameters and average blood velocities for three different types of blood vessels are given in the table below. For each of these blood vessels, estimate the length required for the mean blood temperature to closely approach the tissue temperature, specifically, to satisfy \( \frac{(T_t - T_{m,o})}{(T_t - T_{m,i})} = 0.05 \). The heat transfer between the vessel wall and surrounding tissue can be approximated by an effective heat transfer coefficient, \( h_t = k_t/D \), where \( k_t = 0.5 \text{ W/m} \cdot \text{K} \).

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Diameter, ( D ) (mm)</th>
<th>Blood Velocity, ( u_m ) (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large artery 大動脈</td>
<td>3</td>
<td>130</td>
</tr>
<tr>
<td>Arteriole 動脈</td>
<td>0.02</td>
<td>3</td>
</tr>
<tr>
<td>Capillary</td>
<td>0.008</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Analysis: Since the tissue temperature is fixed and heat transfer between the blood vessel wall and the tissue can be represented by an effective heat transfer coefficient, Equation 8.45a is applicable, with the “free stream” temperature equal to the tissue temperature, \( T_r \). This equation can be used to find the required length \( L \), since \( A_i = \pi DL \). However, we must first find \( \overline{U} \), which requires knowledge of the heat transfer coefficient for the blood flow, \( h_b \). Taking the large artery as an example, the Reynolds number is given by

\[
Re_D = \frac{\rho u_m D}{\mu} = \frac{993 \text{ kg/m}^3 \times 130 \times 10^{-3} \text{ m/s} \times 3 \times 10^{-3} \text{ m}}{695 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 557
\]

so the flow is laminar. Since the other blood vessels have smaller diameters and velocities, their flows will also be laminar. Because we don’t yet know the length of the vessel, we don’t know whether the flow becomes fully developed. However, we will begin by assuming fully developed conditions. Moreover, because the situation is neither one of constant surface temperature nor constant surface heat flux, we will estimate the required length by approximating the Nusselt number as \( Nu \approx 4 \), in which case \( h_b = 4k_b/D \). Neglecting the thermal resistance of the vessel wall, for the large artery

\[
\frac{1}{\overline{U}} = \frac{1}{h_b} + \frac{1}{h_i} = \frac{D}{4k_b} + \frac{D}{k_i} = \frac{3 \times 10^{-3} \text{ m}}{4 \times 0.628 \text{ W/m} \cdot \text{K}} + \frac{3 \times 10^{-3} \text{ m}}{0.5 \text{ W/m} \cdot \text{K}} = 7.2 \times 10^{-3} \text{ m}^2 \cdot \text{K/W}
\]
\[ \bar{U} = 140 \text{ W/m}^2 \cdot \text{K} \]

The length can then be found by solving Equation 8.45a, with \( m = \rho u_m \pi D^2/4 \):

\[
L = -\frac{\rho u_m D c_p}{4 \bar{U}} \ln \left( \frac{T_t - T_{m,\rho}}{T_t - T_{m,f}} \right)
\]

\[
= -\frac{993 \text{ kg/m}^3 \times 130 \times 10^{-3} \text{ m/s} \times 3 \times 10^{-3} \text{ m} \times 4178 \text{ J/kg} \cdot \text{K}}{4 \times 140 \text{ W/m}^2 \cdot \text{K}} \ln(0.05)
\]

\[
= 8.7 \text{ m}
\]

We can now test the assumption that the flow is hydrodynamically and thermally fully developed, using Equations 8.3 and 8.23:

\[
x_{fd,h} = 0.05 Re_D D = 0.05 \times 557 \times 3 \times 10^{-3} \text{ m} = 0.08 \text{ m}
\]

\[
x_{fd,f} = x_{fd,h} Pr = 0.08 \text{ m} \times 4.62 = 0.4 \text{ m}
\]

The flow would indeed be fully developed well within the length of 8.7 m. The calculations can be repeated for the other two cases and are tabulated below.

<table>
<thead>
<tr>
<th>Vessel</th>
<th>( Re_D )</th>
<th>( \bar{U} ) (W/m² · K)</th>
<th>( L ) (m)</th>
<th>( x_{fd,h} ) (m)</th>
<th>( x_{fd,f} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large artery</td>
<td>557</td>
<td>140</td>
<td>8.7</td>
<td>0.08</td>
<td>0.4</td>
</tr>
<tr>
<td>Arteriole</td>
<td>0.086</td>
<td>21,000</td>
<td>8.9 \times 10^{-6}</td>
<td>9 \times 10^{-8}</td>
<td>4 \times 10^{-7}</td>
</tr>
<tr>
<td>Capillary</td>
<td>0.0080</td>
<td>52,000</td>
<td>3.3 \times 10^{-7}</td>
<td>3 \times 10^{-9}</td>
<td>1 \times 10^{-8}</td>
</tr>
</tbody>
</table>
– Thermal Entry Length:

• Average Nusselt Number for Laminar Flow in a Circular Tube with Uniform Surface Temperature:

• Combined Entry Length (Sieder and Tate 速度場與溫度場同時發展):

\[
\left( \frac{Re_D Pr}{L/D} \right)^{1/3} (\mu / \mu_s)^{0.14} < 2: \quad \overline{Nu_D} = 3.66
\]

\[
\left( \frac{Re_D Pr}{L/D} \right)^{1/3} (\mu / \mu_s)^{0.14} > 2: \quad \overline{Nu_D} = 1.86 \left( \frac{Re_D Pr}{L/D} \right)^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}
\]

• Thermal Entry Length (Kays 速度場已發展為拋物線之型態):

\[
\overline{Nu_D} = 3.66 + \frac{0.0668(D/L)Re_D Pr}{1 + 0.04[(D/L)Re_D Pr]^{2/3}}
\]
Effect of the Entry Region

- The manner in which the Nusselt decays from inlet to fully developed conditions for laminar flow depends on the nature of thermal and velocity boundary layer development in the entry region, as well as the surface thermal condition.

Laminar flow in a circular tube

- Combined Entry Length:
  Thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet.
8.5 Turbulent Flow in Circular Tubes

- **Turbulent Flow in a Circular Tube:**
  
  - For a smooth surface and fully turbulent conditions \((\text{Re}_D > 10,000)\), the **Dittus–Boelter equation** may be used as a first approximation:

    \[
    Nu_D = 0.023 \text{Re}_D^{4/5} \Pr^n \\
    \begin{cases} 
      n = 0.3 & (T_s < T_m) \\
      n = 0.4 & (T_s > T_m) 
    \end{cases}
    \]

  - The effects of wall roughness and transitional flow conditions \((\text{Re}_D > 3000)\) may be considered by using the **Gnielinski correlation**:

    \[
    Nu_D = \frac{(f/8)(\text{Re}_D - 1000)\Pr}{1 + 12.7(f/8)^{1/2}(\Pr^{2/3} - 1)}
    \]

    Smooth surface \((e = 0)\): \[f = (0.790 \ln \text{Re}_D - 1.64)^{-2}\]

    Surface of roughness \(e > 0\): \[f \to \text{Figure 8.3}\]
• Average Nusselt Number for Turbulent Flow in a Circular Tube:

  – Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can be neglected.

  – For long tubes \((L/D > 60)\):
    \[
    \bar{Nu}_D \approx Nu_{D,fd}
    \]

  – For short tubes \((L/D < 60)\):
    \[
    \frac{\bar{Nu}_D}{Nu_{D,fd}} \approx 1 + \frac{C}{(L/D)^m} \quad C \approx 1 \quad m \approx 2/3
    \]
EXAMPLE 8.6

Hot air flows with a mass rate of \( \dot{m} = 0.050 \text{ kg/s} \) through an uninsulated sheet metal duct of diameter \( D = 0.15 \text{ m} \), which is in the crawlspace of a house. The hot air enters at 103°C and, after a distance of \( L = 5 \text{ m} \), cools to 77°C. The heat transfer coefficient between the duct outer surface and the ambient air at \( T_\infty = 0°C \) is known to be \( h_o = 6 \text{ W/m}^2 \cdot \text{K} \).

1. Calculate the heat loss (W) from the duct over the length \( L \).
2. Determine the heat flux and the duct surface temperature at \( x = L \).
1. From the energy balance for the entire tube, Equation 8.34,

\[ q = \dot{m}c_p (T_{m,L} - T_{m,0}) \]

\[ q = 0.05 \text{ kg/s} \times 1010 \text{ J/kg} \cdot \text{K} (77 - 103) \text{°C} = -1313 \text{ W} \]

2. An expression for the heat flux at \( x = L \) may be inferred from the resistance network

\[ \frac{T_{m,L}}{q_x(L)} \rightarrow \frac{T_y(L)}{1/h_x(L)} \rightarrow \frac{T_w}{1/h_o} \]

where \( h_x(L) \) is the inside convection heat transfer coefficient at \( x = L \). Hence

\[ q_x(L) = \frac{T_{m,L} - T_w}{[1/h_x(L)] + (1/h_o)} \]

The inside convection coefficient may be obtained from knowledge of the Reynolds number. From Equation 8.6

\[ Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.05 \text{ kg/s}}{\pi \times 0.15 \text{ m} \times 208.2 \times 10^{-7} \text{ N} \cdot \text{s/m}^2} = 20,384 \]
Hence the flow is turbulent. Moreover, with \((L/D) = (5/0.15) = 33.3\), it is reasonable to assume fully developed conditions at \(x = L\). Hence from Equation 8.60, with \(n = 0.3\),

\[
Nu_D = \frac{h_x(L)D}{k} = 0.023 \, Re_D^{4/5} \, Pr^{0.3} = 0.023(20,384)^{4/5} (0.70)^{0.3} = 57.9
\]

\[
h_x(L) = Nu_D \frac{k}{D} = 57.9 \frac{0.030 \, \text{W/m} \cdot \text{K}}{0.15 \, \text{m}} = 11.6 \, \text{W/m}^2 \cdot \text{K}
\]

Hence

\[
q''(L) = \frac{(77 - 0) \, ^\circ \text{C}}{[(1/11.6) + (1/6.0)] \, \text{m}^2 \cdot \text{K}/\text{W}} = 304.5 \, \text{W/m}^2
\]

Referring back to the network, it also follows that

\[
q''(L) = \frac{T_{m,L} - T_{s,L}}{1/h_x(L)}
\]

in which case

\[
T_{s,L} = T_{m,L} - \frac{q''(L)}{h_x(L)} = 77^\circ \text{C} - \frac{304.5 \, \text{W/m}^2}{11.6 \, \text{W/m}^2 \cdot \text{K}} = 50.7^\circ \text{C}
\]
8.6 Noncircular Tubes

- Entry lengths depend on whether the flow is laminar or turbulent, which, in turn, depend on Reynolds number.

\[
\text{Re}_D \equiv \frac{\rho u_m D_h}{\mu}
\]

The hydraulic diameter is defined as

\[
D_h \equiv \frac{4A_c}{P}
\]

in which case,

\[
\text{Re}_D \equiv \frac{\rho u_m D_h}{\mu} = \left(\frac{\rho u_m A_c}{\mu} \right) D_h = \frac{\dot{m}}{\mu} \frac{D_h}{A_c} = \frac{4\dot{m}}{P \mu}
\]

- **Laminar Flow:**

  The local Nusselt number is a constant whose value (Table 8.1) depends on the surface thermal condition \((T_s \text{ or } q_s')\) and the duct aspect ratio.

- **Turbulent Flow:**

  As a first approximation, the Dittus-Boelter or Gnielinski correlation may be used with the hydraulic diameter, irrespective of the surface thermal condition.
– Turbulent Flow:

As a first approximation, correlations for a circular tube may be used with $D$ replaced by $D_h$.

- When determining $\overline{Nu_D}$ for any tube geometry or flow condition, all properties are to be evaluated at

$$\overline{T_m} \equiv \left( T_{m,i} + T_{m,o} \right) / 2$$

Why do solutions to internal flow problems often require iteration (迭代)?
<table>
<thead>
<tr>
<th>Cross Section</th>
<th>( \frac{b}{a} )</th>
<th>( N_u_D = \frac{hD_u}{k} )</th>
<th>(Uniform ( q''_v ))</th>
<th>(Uniform ( T_v ))</th>
<th>( f Re_Du )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>1.0</td>
<td>4.36</td>
<td>3.66</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>1.43</td>
<td>3.61</td>
<td>2.98</td>
<td>59</td>
<td></td>
</tr>
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<td>Rectangle</td>
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<td>3.08</td>
<td>59</td>
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</tr>
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<td>3.96</td>
<td>69</td>
<td></td>
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<td>Rectangle</td>
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<td>5.33</td>
<td>4.44</td>
<td>73</td>
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</tr>
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<td>Rectangle</td>
<td>8.0</td>
<td>6.49</td>
<td>5.60</td>
<td>82</td>
<td></td>
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<tr>
<td>Infinite</td>
<td>( \infty )</td>
<td>8.23</td>
<td>7.54</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>Heated</td>
<td>( \infty )</td>
<td>5.39</td>
<td>4.86</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>Insulated</td>
<td>2.0</td>
<td>3.11</td>
<td>2.49</td>
<td>53</td>
<td></td>
</tr>
</tbody>
</table>

The Concentric Tube Annulus (略)

- Fluid flow through region formed by concentric tubes.

- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.

- Surface thermal conditions may be characterized by uniform temperature \( (T_{s,i}, T_{s,o}) \) or uniform heat flux \( (q_i'', q_o'') \).

- Convection coefficients are associated with each surface, where

\[
q_i'' = h_i \left( T_{s,i} - T_m \right) \quad q_o'' = h_o \left( T_{s,o} - T_m \right)
\]
\[ Nu_i \equiv \frac{h_i D_h}{k} \quad \text{and} \quad Nu_o \equiv \frac{h_o D_h}{k} \]

where \( D_h = D_o - D_i \)

- **Fully Developed Laminar Flow**

  Nusselt numbers depend on \( D_i / D_o \) and surface thermal conditions (Tables 8.2, 8.3)

- **Fully Developed Turbulent Flow**

  Correlations for a circular tube may be used with \( D \) replaced by \( D_h \).
<table>
<thead>
<tr>
<th>$D_i/D_o$</th>
<th>$Nu_i$</th>
<th>$Nu_o$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>3.66</td>
<td>See Equation 8.55</td>
</tr>
<tr>
<td>0.05</td>
<td>17.46</td>
<td>4.06</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>11.56</td>
<td>4.11</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>7.37</td>
<td>4.23</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>5.74</td>
<td>4.43</td>
<td></td>
</tr>
<tr>
<td>$\approx 1.00$</td>
<td>4.86</td>
<td>4.86</td>
<td>See Table 8.1, $b/a \to \infty$</td>
</tr>
</tbody>
</table>

Table 8.3 Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

<table>
<thead>
<tr>
<th>$D_i/D_o$</th>
<th>$Nu_{ii}$</th>
<th>$Nu_{oo}$</th>
<th>$\theta_i^*$</th>
<th>$\theta_o^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>4.364</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>17.81</td>
<td>4.792</td>
<td>2.18</td>
<td>0.0294</td>
</tr>
<tr>
<td>0.10</td>
<td>11.91</td>
<td>4.834</td>
<td>1.383</td>
<td>0.0562</td>
</tr>
<tr>
<td>0.20</td>
<td>8.499</td>
<td>4.833</td>
<td>0.905</td>
<td>0.1041</td>
</tr>
<tr>
<td>0.40</td>
<td>6.583</td>
<td>4.979</td>
<td>0.603</td>
<td>0.1823</td>
</tr>
<tr>
<td>0.60</td>
<td>5.912</td>
<td>5.099</td>
<td>0.473</td>
<td>0.2455</td>
</tr>
<tr>
<td>0.80</td>
<td>5.58</td>
<td>5.24</td>
<td>0.401</td>
<td>0.299</td>
</tr>
<tr>
<td>1.00</td>
<td>5.385</td>
<td>5.385</td>
<td>0.346</td>
<td>0.346</td>
</tr>
</tbody>
</table>


\[
Nu_i = \frac{Nu_{ii}}{1 - (q_o''/q_i'')\theta_i^*} \quad Nu_o = \frac{Nu_{oo}}{1 - (q_i''/q_o'')\theta_o^*}
\]
Figure 8.12 Internal flow heat transfer enhancement schemes: (a) longitudinal section and end view of coil-spring wire insert, (b) longitudinal section and cross-sectional view of twisted tape insert, (c) cut-away section and end view of longitudinal fins, and (d) longitudinal section and end view of helical ribs.
Figure 8.13  Schematic of helically coiled tube and secondary flow in enlarged cross-sectional view.
The secondary flow increases friction losses and heat transfer rates. In addition, the secondary flow decreases entrance lengths and reduces the difference between laminar and turbulent heat transfer rates, relative to the straight tube cases considered previously in this chapter. Pressure drops and heat transfer rates exhibit little dependence on the coil pitch, \( S \). The critical Reynolds number corresponding to the onset of turbulence for the helical tube, \( Re_{D,c,h} \) is

\[
Re_{D,c,h} = Re_{D,c}[1 + 12(D/C)^{0.5}] \quad (8.74)
\]

where \( Re_{D,c} \) is given in Equation 8.2 and \( C \) is defined in Figure 8.13. Strong secondary flow associated with tightly wound helically coiled tubes delays the transition to turbulence.
For fully developed laminar flow with $C/D \geq 3$, the friction factor is

\[ f = \frac{64}{Re_D} \quad Re_D(D/C)^{1/2} \leq 30 \quad (8.19) \]

\[ f = \frac{27}{Re_D^{0.725}} (D/C)^{0.1375} \quad 30 \leq Re_D(D/C)^{1/2} \leq 300 \quad (8.75a) \]

\[ f = \frac{7.2}{Re_D^{0.5}} (D/C)^{0.25} \quad 300 \leq Re_D(D/C)^{1/2} \quad (8.75b) \]

For cases where $C/D \leq 3$, recommendations provided in [29] should be followed. The heat transfer coefficient for use in Equation 8.27 may be evaluated from a correlation of the form

\[ Nu_D = \left[ \left(3.66 + \frac{4.343}{a}\right)^{1/3} + 1.158 \left(\frac{Re_D(D/C)^{1/2}}{b}\right)^{3/2}\left(\frac{\mu_t}{\mu_i}\right)^{0.14} \right]^{1/3} (8.76) \]

where

\[ a = \left(1 + \frac{927(C/D)}{Re_D^2 Pr}\right) \quad \text{and} \quad b = 1 + \frac{0.477}{Pr} \quad (8.77a,b) \]

\[ \begin{bmatrix} 0.005 \leq Pr \leq 1600 \hfill \\
1 \leq Re_D(D/C)^{1/2} \leq 1000 \hfill \\
\end{bmatrix} \]
Equations in Chapters 6-8 should not be used for gases when $D_h/\lambda_{mfp} \leq 10$ and should be used with caution for liquids when $D_h \leq 1 \, \mu m$. 
Example 8.7

Combinatorial chemistry and biology are used in the pharmaceutical and biotechnology industries to reduce the time and cost associated with producing new drugs. Scientists desire to create large populations of molecules, or libraries, that can be subsequently screened en masse. Producing vast libraries increases the probability that novel compounds of significant therapeutic value will be discovered. A crucial variable in producing new compounds is the temperature at which the reactants are processed.

A microreactor chip is fabricated by first coating a 1-mm-thick glass microscope slide with a photoresist material. Lines are subsequently etched into the photoresist and a second glass plate is attached to the top of the structure, resulting in multiple parallel channels of rectangular cross section that are $a = 40 \ \mu m$ deep, $b = 160 \ \mu m$ wide, and $L = 20 \ mm$ long. The spacing between channels is $s = 40 \ \mu m$, so that $N = L/(w + s) = 100$ channels are present within the $20 \ mm \times 20 \ mm$ microreactor. A mixture of two reactants, both initially at $T_{m,1} = 5^\circ C$, is introduced into each channel, and the edges of the chip are maintained at temperatures $T_1 = 125^\circ C$ and $T_2 = 25^\circ C$ so that the reactants in each channel are subject to a unique processing temperature. Flow is induced through the structure by applying an overall pressure difference of $\Delta P = 500 \ kPa$. The reactants and the product of reaction have thermophysical properties similar to ethylene glycol. Estimate the time that elapses for the entering reactants to come within $1^\circ C$ of the desired processing temperature.
**Reactants A and B**  
**T_{m,i} = 5\,^\circ C**

- **b = 160 \, \mu m**
- **a = 40 \, \mu m**
- **s = 40 \, \mu m**

**Glass**

**Photoresist material**

**Microchannel array**

**T_1 = 125\,^\circ C**  
**L = 20 mm**

**Glass**

**T_2 = 25\,^\circ C**
**Analysis:** We will bracket the heat transfer and fluid flow behavior by evaluating the performance at the extreme processing temperatures. The flow of reactants is induced by the applied pressure difference between the inlet and outlet of the microreactor. Because of the large variation of the viscosity with temperature, we expect the flow rate that is associated with the highest processing temperature to be the largest.

The perimeter of each microchannel is

$$P = 2a + 2b = 2 \times 40 \times 10^{-6} \text{ m} + 2 \times 160 \times 10^{-6} \text{ m} = 0.4 \times 10^{-3} \text{ m}$$

and the hydraulic diameter of each microchannel is found from Equation 8.66 as

$$D_h = \frac{4A_c}{P} = \frac{4ab}{P} = \frac{4 \times 40 \times 10^{-6} \text{ m} \times 160 \times 10^{-6} \text{ m}}{0.4 \times 10^{-3} \text{ m}} = 64 \times 10^{-6} \text{ m}$$
We begin by assuming a relatively short entrance length, to be verified later, so the flow rate may be estimated by using the friction factor for fully developed conditions. From Table 8.1 for \( b/a = 4, f = 73/Re_D \). Substituting this expression into Equation 8.22a, rearranging terms, and using properties at \( T = 125^\circ C \) (in this equation and those following) results in

\[
u_m = \frac{2}{73} \frac{D_h^2 \Delta p}{\mu L} = \frac{2}{73} \times \frac{(64 \times 10^{-6} \text{ m})^2 \times 500 \times 10^3 \text{ N/m}^2}{0.427 \times 10^{-2} \text{ N} \cdot \text{s/m}^2 \times 20 \times 10^{-3} \text{ m}} = 0.657 \text{ m/s}
\]

Hence, the Reynolds number is

\[
Re_D = \frac{u_m D_h \rho}{\mu} = \frac{0.657 \text{ m/s} \times 64 \times 10^{-6} \text{ m} \times 1085 \text{ kg/m}^3}{0.427 \times 10^{-2} \text{ N} \cdot \text{s/m}^2} = 10.7
\]

and the flow is deep in the laminar regime. Equation 8.3 may be used to determine the hydrodynamic entrance length, which is

\[
x_{ld,h} = 0.05D_h Re_D = 0.05 \times 64 \times 10^{-6} \text{ m} \times 10.7 = 34.2 \times 10^{-6} \text{ m}
\]
and the thermal entrance length may be obtained from Equation 8.23, yielding

\[ x_{\text{fd},l} = x_{\text{fd},l} Pr = 34.2 \times 10^{-6} \text{ m} \times 45.2 = 1.55 \times 10^{-3} \text{ m} \]

Both entrance lengths occupy less than 10\% of the total microchannel length, \( L = 20 \text{ mm} \). Therefore, use of fully developed values of \( f \) are justified, and the mass flow rate for the \( T = 125^\circ\text{C} \) microchannel is

\[ \dot{m} = \rho A_c u_m = \rho ab u_m = 1085 \text{ kg/m}^3 \times 40 \times 10^{-6} \text{ m} \times 160 \times 10^{-6} \text{ m} \times 0.657 \text{ m/s} \]
\[ = 4.56 \times 10^{-6} \text{ kg/s} \]

Equation 8.42 may now be used to determine the distance from the entrance of the microchannel to the location, \( x_c \), where \( T_{mc} = 124^\circ\text{C} \), that is, within 1\% of the surface temperature. The average heat transfer coefficient, \( \bar{h} \), is replaced by the fully developed value of the heat transfer coefficient, \( h \), because of the relatively short thermal entrance length. From Table 8.1, we see that for \( b/a = 4, Nu_D = hD_h/k = 4.44 \). Therefore,

\[ \bar{h} = h = Nu_D \frac{k}{D_h} = 4.44 \times \frac{0.261 \text{ W/m} \cdot \text{K}}{64 \times 10^{-6} \text{ m}} = 1.81 \times 10^4 \text{ W/m}^2 \cdot \text{K} \]
As expected from our discussion of microscale flows, the convection coefficient is extremely large.

Rearranging Equation 8.42 yields

\[ x_c = \frac{mc_p}{Ph} \ln \left( \frac{T_s - T_{m,c}}{T_s - T_{m,i}} \right) = \frac{4.56 \times 10^{-6} \text{ kg/s} \times 2583 \text{ J/kg} \cdot \text{K}}{0.4 \times 10^{-3} \text{ m} \times 1.81 \times 10^4 \text{ W/m}^2 \cdot \text{K}} \ln \left( \frac{(125 - 5) \text{°C}}{(125 - 124) \text{°C}} \right) \]

\[ = 7.79 \times 10^{-3} \text{ m} \]

Therefore, the time needed for the reactant to reach a mean temperature that is within 1°C of the processing temperature is

\[ t_c = \frac{x_c}{u_m} = 7.79 \times 10^{-3} \text{ m} / 0.657 \text{ m/s} = 0.012 \text{ s} \]

Repeating the calculations for the microchannel associated with the smallest processing temperature of 25°C yields \( u_m = 0.0995 \text{ m/s} \), \( Re_D = 0.253 \), \( x_{f,d} = 8.09 \times 10^{-7} \text{ m} \), \( x_{f,d,h} = 0.218 \times 10^{-3} \text{ m} \), \( h = 1.71 \times 10^4 \text{ W/m}^2 \cdot \text{K} \), \( x_c = 0.73 \times 10^{-3} \text{ m} \), and \( t_c = 0.007 \text{ s} \).
Problem 8.43: For an air passage used to cool a gas turbine vane, calculate the air outlet temperature and heat removed from the vane.

**KNOWN:** Diameter and length of copper tubing. Temperature of collector plate to which tubing is soldered. Water inlet temperature and flow rate.
**FIND:** (a) Outlet temperature of the air coolant for the prescribed conditions and (b) Compute and plot the air outlet temperature \( T_{m,o} \) as a function of flow rate, \( 0.1 \leq \dot{m} \leq 0.6 \text{ kg/h} \). Compare this result with those for vanes having passage diameters of 2 and 4 mm.

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation.

**PROPERTIES:** *Table A.4*, Air (assume \( T_m = 780 \text{ K}, \ 1 \text{ atm} \)): \( c_p = 1094 \text{ J/kg} \cdot \text{K}, \ k = 0.0563 \text{ W/m} \cdot \text{K}, \ \mu = 363.7 \times 10^{-7} \text{ N} \cdot \text{s/m}^2, \ Pr = 0.706; \ (T_s = 650^\circ\text{C} = 923 \text{ K}, \ 1 \text{ atm}) : \ \mu = 404.2 \times 10^{-7} \text{ N} \cdot \text{s/m}^2.*
ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.41b,

\[
\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp \left( -\frac{PLh}{mc_p} \right)
\]

where \( P = \pi D \). For flow in a circular passage,

\[
Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.18 \text{ kg/h} \times (1/3600 \text{ s/h})}{\pi (0.003 \text{ m}) 363.7 \times 10^{-7} \text{ N·s/m}^2} = 584.
\]

The flow is laminar, and since \( L/D = 75 \text{ mm}/3 \text{ mm} = 25 \), the Sieder-Tate correlation including combined entry length yields

\[
\text{Nu}_D = \frac{\bar{h}D}{k} = 1.86 \left( \frac{Re_D Pr}{L/D} \right)^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}
\]

\[
\bar{h} = \frac{0.0563 \text{ W/m·K}}{0.003 \text{ m}} 1.86 \left( \frac{584 \times 0.706}{25} \right)^{1/3} \left( \frac{363.7 \times 10^{-7}}{404.2 \times 10^{-7}} \right)^{0.14} = 87.5 \text{ W/m}^2 \cdot \text{K}.
\]

Hence, the air outlet temperature is

\[
\frac{650 - T_{m,o}}{(650 - 427)^\circC} = \exp \left( -\frac{\pi(0.003 \text{ m}) \times 0.075 \text{ m} \times 87.5 \text{ W/m}^2 \cdot \text{K}}{(0.18/3600) \text{ kg/s} \times 1094 \text{ J/kg·K}} \right)
\]

\( T_{m,o} = 578^\circ \text{C} \)
(b) Using the *IHT Correlations Tool, Internal Flow*, for *Laminar Flow* with *combined entry length*, along with the energy balance and rate equations above, the outlet temperature $T_{m,o}$ was calculated as a function of flow rate for diameters of $D = 2$, $3$ and $4$ mm. The plot below shows that $T_{m,o}$ decreases nearly linearly with increasing flow rate, but is independent of passage diameter.
Based upon the calculation for $T_{m,o} = 578^\circ C$, $T_m = 775 \text{ K}$ which is in good agreement with our assumption to evaluate the thermophysical properties.

Why is $T_{m,o}$ independent of $D$? From Eq. (3), note that $\bar{h}$ is inversely proportional to $D$, $\bar{h} \sim D^{-1}$. From Eq. (1), note that on the right-hand side the product $P \cdot \bar{h}$ will be independent of $D$. Hence, $T_{m,o}$ will depend only on $\dot{m}$. This is, of course, a consequence of the laminar flow condition and will not be the same for turbulent flow.
Problem 8.52: Determine effect of ambient air temperature and wind velocity on temperature at which oven gases are discharged from a stack.

**KNOWN:** Thin-walled, tall stack discharging exhaust gases from an oven into the environment.
**FIND:** (a) Outlet gas and stack surface temperatures, $T_{m,o}$ and $T_{s,o}$, for $V=5$ m/s and $T_\infty = 4^\circ C$; (b) Effect of wind temperature and velocity on $T_{m,o}$.

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible wall thermal resistance, (3) Exhaust gas properties approximately those of atmospheric air, (4) Negligible radiative exchange with surroundings, (5) Ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed flow, (7) Constant properties.

**PROPERTIES:** Table A.4, air (assume $T_{m,o} = 773$ K, $\overline{T}_m = 823$ K, 1 atm): $c_p = 1104$ J/kg·K, $\mu = 376.4 \times 10^{-7}$ N·s/m$^2$, $k = 0.0584$ W/m·K, $Pr = 0.712$; Table A.4, air (assume $T_s = 523$ K, $T_\infty = 4^\circ C = 277$ K, $T_f = 400$ K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m$^2$/s, $k = 0.0338$ W/m·K, $Pr = 0.690$. 
**ANALYSIS:** (a) From Eq. 8.45a,

\[ T_{m,o} = T_\infty - \left( T_\infty - T_{m,i} \right) \exp \left[ -\frac{PL}{mc_p} \bar{U} \right] \]

\[ U = \frac{1}{\left( \frac{1}{h_i} + \frac{1}{h_o} \right)} \]

where \( h_i \) and \( h_o \) are average coefficients for internal and external flow, respectively.

*Internal flow:* With a Reynolds number of

\[ \text{Re}_{D_i} = \frac{4 \dot{m}}{\pi D \mu} = \frac{4 \times 0.5 \text{ kg/s}}{\pi \times 0.5 \text{ m} \times 376.4 \times 10^{-7} \text{ N \cdot s/m}^2} = 33,827 \]
The flow is turbulent, and assuming fully developed conditions throughout the stack, the Dittus-Boelter correlation may be used to determine $h_i$.

$$\frac{\bar{h}_i}{D} = \frac{\bar{h}_i D}{k} = 0.023 \frac{Re^{4/5} Pr^{0.3}}{D_i}$$

$$\bar{h}_i = \frac{58.4 \times 10^{-3} \text{ W/m} \cdot \text{K}}{0.5 \text{ m}} \times 0.023 (33,827)^{4/5} (0.712)^{0.3} = 10.2 \text{ W/m}^2 \cdot \text{K}$$

**External flow:** Working with the Churchill/Bernstein correlation and

$$Re_{Do} = \frac{VD}{\nu} = \frac{5 \text{ m/s} \times 0.5 \text{ m}}{26.41 \times 10^{-6} \text{ m}^2/\text{s}} = 94,660$$

$$\bar{Nu}_D = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 205$$
The outlet stack surface temperature can be determined from a local surface energy balance of the form,

$$h_i(T_{m,o} - T_{s,o}) = h_o(T_{s,o} - T_\infty),$$

which yields

$$T_{s,o} = \frac{h_i T_{m,o} + h_o T_\infty}{h_i + h_o} = \frac{(10.2 \times 543 + 13.9 \times 4) \text{ W/m}^2}{(10.2 + 13.9) \text{ W/m}^2 \cdot \text{K}} = 232^\circ \text{C}$$

Hence,

$$\overline{h_o} = (0.0338 \text{ W/m} \cdot \text{K}/0.5 \text{ m}) \times 205 = 13.9 \text{ W/m}^2 \cdot \text{K}$$

The outlet gas temperature is then

$$T_{m,o} = 4^\circ \text{C} - (4 - 600)^\circ \text{C} \exp \left[ -\frac{\pi \times 0.5 \times 6}{0.5 \text{ kg/s} \times 1104 \text{ J/kg K}} \left( \frac{1}{1/10.2 + 1/13.9} \text{ W/m}^2 \cdot \text{K} \right) \right] = 543^\circ \text{C}$$
b) The effects of the air temperature and velocity are as follows

Due to the elevated temperatures of the gas, the variation in ambient temperature has only a small effect on the gas exit temperature. However, the effect of the freestream velocity is more pronounced. Discharge temperatures of approximately 530 and 560°C would be representative of cold/windy and warm/still atmospheric conditions, respectively.

**COMMENTS:** If there are constituents in the gas discharge that condense or precipitate out at temperatures below $T_{s,o}$, related operating conditions should be avoided.
Homework 9

- 8.24
- 8.74
- 8.75